

On the Martingale Property of Certain Local Martingales

¹ *Imperial College London, UK*

E-mail: a.mijatovic@imperial.ac.uk

² *Universität Ulm, Germany*

E-mail: mikhail.urusov@uni-ulm.de

The stochastic exponential Z of a continuous local martingale M is itself a continuous local martingale. We give a necessary and sufficient condition for the process Z to be a true martingale and for the process Z to be a uniformly integrable martingale in the case where $M_t = \int_0^t b(Y_u) dW_u$, the process Y is a one-dimensional diffusion, and the process W is a Brownian motion. These conditions are deterministic and expressed only in terms of the function b and the drift and diffusion coefficients of Y . Such questions arise in stochastic analysis and its applications whenever one needs to perform a (locally) absolutely continuous measure change. For instance, one can characterize several notions of no-arbitrage and examine relations between them, or characterize one-dimensional diffusion models with bubbles using our results.