A. GORITSKY

Inertial Manifolds and Gap Property for Dissipative Hyperbolic Equations

Department of Mechanics and Mathematics, Moscow State University,
E-mail: goritsky@mech.math.msu.su

Consider the dissipative semilinear hyperbolic equation

$$u_{tt} + 2\gamma\, u_t = \Delta u + f(u) + g(x), \qquad u\big|_{\partial\Omega} = 0, \tag{1}$$

$$u\big|_{t=0} = u_0(x) \in H_0^1(\Omega), \qquad u_t\big|_{t=0} = p_0(x) \in L_2(\Omega). \tag{2}$$

in a bounded domain Ω. Here $\gamma > 0$ is a coefficient of weak dissipation, $g(x) \in L_2(\Omega)$ is an external force, and the nonlinearity $f(u)$ satisfies the global Lipschitz condition $|f(u_1) - f(u_2)| \leqslant l\,|u_1 - u_2|$.

Let λ_k, $0 < \lambda_1 < \lambda_2 \leqslant \cdots \to +\infty$, be eigenvalues of the operator $-\Delta$ in the domain Ω under the Dirichlet boundary conditions. Suppose $\lambda_{N+1} - \lambda_N > 4l$ for some N and γ is sufficiently large. Then, in the phase space $H_0^1(\Omega) \times L_2(\Omega)$, there exists an N-dimensional inertial manifold M that exponentially attracts all the solutions of problem (1), (2).