N. TARKHANOV

Numerical Experiments for the Riemann Hypothesis

Institute of Mathematics, Potsdam, Germany
E-mail: tarkhanov@math.uni-potsdam.de

The Riemann zeta function $\zeta(z)$ is defined by the convergent series

$$\zeta(z) := \sum_{n=1}^{\infty} \frac{1}{n^z}$$

for all complex z satisfying $\Re z > 1$. It extends to a meromorphic function in the whole complex plane with the only pole at $z = 1$ which is simple. This function vanishes at $z = -2, -4, -6, \ldots$, the other zeros of $\zeta(z)$ are known to lie in the critical strip $0 < \Re z < 1$. Riemann conjectured (1859) that all zeros of $\zeta(z)$ in the critical strip belong to the line $\Re z = 1/2$.

Theorem 1 *The Riemann hypothesis holds if and only if* $\lim_{n \to \infty} \sqrt[n]{|c_n|} = 1$, *where*

$$c_n = 3 \int_{-1}^{1} \frac{1}{\zeta\left(\frac{3}{4} + \frac{1}{\pi} \arctan x\right)} \left(\frac{2 - ix}{i + 2x}\right)^n \frac{dx}{(i + 2x)(2 - ix)}. \tag{1}$$

See [1, 2].

For computations we used the mathematics programs “Mathematica”, “Maple” and “Matlab”.

[1] L. Aizenberg et al., *One computational approach in support of the Riemann hypothesis*, Computers and Mathematics with Applications **37** (1999), 87–94.

[2] M. Albinus, *Numerische Experimente zur Riemannschen Hypothese*, Institut für Mathematik, Universität Potsdam, 2010.